1 Definitions

Define random variables

- s^* denoting skill
- ε denoting measurement error, with $\mathbb{E}[\varepsilon] = 0$, ε independent of all other random variables included in the model
- s_s^* denoting self-assessed skill

Then we define performance p as

$$p \coloneqq s^* + \varepsilon \tag{1}$$

and overconfidence oc^* as

$$oc^* \coloneqq s_s^* - s^* \tag{2}$$

and expected performance p_e as

$$p_e \coloneqq s^* + oc^* \tag{3}$$

Overconfidence oc^* is measured by overestimation oe defined as

$$oe \coloneqq p_e - p \tag{4}$$

2 Theorems

Theorem 1:

$$oe = oc^* - \varepsilon \tag{5}$$

Proof 1: From eq. 2 and 3 it follows that $p_e = s_s^*$ and further from eq. 3 and 1 we see

$$\begin{aligned} oe &= p_e - p && (6)\\ &= (s^* + oc^*) - (s^* + \varepsilon) && (7)\\ &= oc^* - \varepsilon && (8) \end{aligned}$$

3 Statistical Models

3.1 Linear Model

Using a linear regression model, the Dunning-Kruger effect can be stated as

$$oc^* = \alpha + \beta_1 s^* + u \tag{9}$$

with $\beta_1 < 0$. Substituting the observable variables and rearranging according to eq. 1 and 6:

$$oe = \alpha + \beta_1 p + u - \varepsilon(1 + \beta_1) \tag{10}$$

3.1.1 Correction

There are different ways to correct for the bias introduced by measurement error:

- Bias correction: use a bias correction formula that takes into account the correlation between performance and the error term
- IV approach: measure performance on a second test (p_2) and compute $\beta_1 = \frac{\mathrm{cov}(oe,p_2)}{\mathrm{cov}(p,p_2)}$.